Reply to "Comment on 'Adiabatic domain wall motion and Landau-Lifshitz damping' "

M. D. Stiles,[1] W. M. Saslow,[2] M. J. Donahue,[3] and A. Zangwill[4]
[1]*Center for Nanoscale Science and Technology, National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8412, USA*
[2]*Department of Physics, Texas A&M University, College Station, Texas 77843-4242, USA*
[3]*Mathematical and Computational Sciences Division, National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8910, USA*
[4]*School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332-0430, USA*
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The Comment by Smith argues that there are fundamental differences between models for current-induced domain wall motion based on either Gilbert damping or the Landau-Lifshitz form. Here, we expand a point mentioned several times in our paper.[1] The two equations of motion are mathematically equivalent and one cannot be correct without the other being correct. The Landau-Lifshitz-Gilbert form of the equation of motion,

$$\dot{\mathbf{M}} = -\gamma_0 \mathbf{M} \times \mathbf{H} + \frac{\alpha}{M_s}\mathbf{M} \times \dot{\mathbf{M}} + v_s(\hat{\mathbf{j}} \cdot \boldsymbol{\nabla})\mathbf{M} - \frac{\beta}{M_s} v_s \mathbf{M} \times (\hat{\mathbf{j}} \cdot \boldsymbol{\nabla})\mathbf{M}, \tag{1}$$

and the Landau-Lifshitz form,

$$\dot{\mathbf{M}} = -\gamma \mathbf{M} \times \mathbf{H} - \frac{\lambda}{M_s}\mathbf{M} \times (\mathbf{M} \times \mathbf{H}) + v_s'(\hat{\mathbf{j}} \cdot \boldsymbol{\nabla})\mathbf{M} - \frac{\beta'}{M_s} v_s' \mathbf{M} \times (\hat{\mathbf{j}} \cdot \boldsymbol{\nabla})\mathbf{M}, \tag{2}$$

describe *exactly* the same time evolution provided that

$$\gamma = \gamma_0/(1+\alpha^2),$$

$$\lambda = \alpha\gamma_0/(1+\alpha^2),$$

$$v_s' = v_s(1+\alpha\beta)/(1+\alpha^2),$$

$$\beta' = (\beta - \alpha)/(1+\alpha\beta). \tag{3}$$

To lowest order in the small parameters α and β, these conditions are reduced to

$$\gamma = \gamma_0,$$

$$\lambda = \alpha\gamma_0,$$

$$v_s' = v_s,$$

$$\beta' = (\beta - \alpha). \tag{4}$$

Here, $\mathbf{M}$ is the magnetization, M_s is the saturation magnetization, $\mathbf{H}$ is the effective field acting on the magnetization, γ_0 and γ are the gyromagnetic ratios, α and λ are the damping parameters, v_s and v_s' are the effective velocities parametrizing the adiabatic spin-transfer torque, $\hat{\mathbf{j}}$ is the direction of the current, and β and β' parametrize the correction to the adiabatic spin-transfer torque.

It is the case that, as pointed out in the discussion near Eq. (4) of the Comment, inserting the same phenomenological spin-transfer torque term into the Landau-Lifshitz and Landau-Lifshitz-Gilbert equations of motion gives equations of motion that predict different motion. However, all of the terms in the equation of motion must be derived in the same framework to tell which is correct.[2] The authors of Ref. 2 have derived these equations of motion using methods that naturally give terms including the time derivative of the magnetization, and their results are in the Landau-Lifshitz-Gilbert form. However, as seen above, they can be recast into an equivalent Landau-Lifshitz form. The two forms differ mainly in their partitioning of terms that depend on both the coupling to the heat bath (damping) and the current (spin-transfer torque). Whether the parameters that describe the motion are "simpler" in one form or the other depends on higher order terms, which have not been determined.

Given that the two equations of motion are equivalent, the choice of one or the other is based on ease of interpretation, convenience, or pedagogy. In our paper, we offer some reasons to consider the Landau-Lifshitz form. One of us has offered additional reasons.[3] In the preceding Comment, Smith offers some reasons to consider the Landau-Lifshitz-Gilbert form.

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